Probability **Sample Final Exam**

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Name:

Instructions:

- All answers must be written clearly.
- You may use a calculator, but you must show all your work in order to receive credit.
- Be sure to erase or cross out any work that you do not want graded.
- If you need extra space, you may use the back sides of the exam pages (if you do, please write me a note so that I know where to look).
- You must include all work to receive full credit.

1. Consider a standard deck of 52 cards. What is the probability of a four of a kind? (This occurs when the cards have denominations a, a, a, a, b.)

2. Consider a roullete wheel consisting of 50 numbers 1 through 50, 0, and 00. If Phan always bets that the outcome will be one of the numbers 1 through 20, what is the probability that

   (a) Phan will lose his first 7 bets,

   (b) his first win will occur on his ninth bet?

3. The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that at most 1 accident will occur in next 2 months?

4. The r.v. X has a mgf given by

$$m_X(t) = \frac{1}{1-t}, \quad t < 1.$$

If u is some unknown number greater than 0, what is $\mathbb{P}(X > 1 + u \mid X > u)$?

5. A manufacturing company sources widgets from three different suppliers (A, B, and C). Based on the company's quality control data, it appears that 3 percent of widgets coming from A are faulty, as are 5 percent of the widgets coming from B, and 2 percent coming from C. Based on recent purchasing records, suppliers A, B, and C supply 30 percent, 20 percent, and 50 percent of the company's stock of widgets, respectively.

   (a) What is the probability that a random widget from the company's stock is faulty?

   (b) Given that a widget is faulty, what is the probability that it came from supplier C?

   (c) Using the definition of independence of events, determine whether the events $F = \{\text{widget is faulty}\}$ and $C = \{\text{widget came from supplier C}\}$ are independent or not.

6. Suppose the joint density function of the random variables X and Y is

$$f(x,y) = \begin{cases} c\,(x+y) & 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Find the value of c.

(b) Compute $\mathbb{P}\left(X^2 + Y^2 \leq 1\right)$

(c) Compute $\mathbb{E}\left[X^2Y\right]$.

7. Suppose X is a normal r.v. with mean 1 and variance 1 and let Y be an independent Poisson r.v. with parameter 2. What is $\mathrm{Var}\left(2X - Y\right)$?

8. Let X be a uniform random variable over $(1, 6)$. Find the moment generating function of X.

9. Suppose X has the following moment generating function

$$m_X(t) = \frac{e^t}{1 - t^2}.$$

Find $\mathbb{E}[X]$. (This distribution is known as the *Laplace* distribution)

10. A person has 100 light bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working light bulb after 525 hours.